**Managing Morning Commute Congestion with a Tradable Credit Scheme Under Commuter Heterogeneity and Market Loss Aversion Behavior**

Mohammad Miralinaghi1,2, Srinivas Peeta1,2,[[1]](#footnote-1)\* , Xiaozheng He1,3, Satish V. Ukkusuri2

1NEXTRANS Center, Purdue University, 3000 Kent Avenue, West Lafayette, IN 47906, USA

2School of Civil Engineering, Purdue University, West Lafayette, IN 47907, USA

3Department of Civil and Environmental Engineering, Rensselaer Polytechnic Institute, Troy NY 12180, USA

**Abstract**

This study analyzes the effect of a tradable credit scheme (TCS) on managing morning commute congestion. It uses the single bottleneck model in a discrete time setting, and classifies commuters into groups based on heterogeneity in terms of value of time, schedule delay penalty and desired arrival time. The TCS consists of group-specific credit allocation and time-varying credit charging schemes. Commuters trade credits in the market based on their initial credit endowments, travel needs, and loss aversion behavior towards purchasing credits in the market. Commuters valuate the charged credits of departure time choice as monetary gains or losses relative to their initial endowments. The existence and uniqueness of equilibrium departure rates, credit price and travel disutility are investigated under this TCS. The system optimal design of the TCS is formulated by applying linear programming duality to derive the TCS parameters that lead to the system optimal solution in terms of the sum of the schedule delay and travel time costs. It is demonstrated that credit price and total value of traded credits approach zero as consumers’ loss aversion increases. Sensitivity analysis is performed to understand the effects of the initial credit allocation scheme on credit price and travel disutility of commuters. It is proved that the initial credit allocation method does not impact the credit price and departure rate of commuters if commuters are equally sensitive to monetary loss and gain. The study findings suggest that if the market loss aversion behavior of commuters is not considered, the system-optimal TCS design can lead to an inequitable scheme where some commuters experience high travel disutilities which results in public opposition toward its implementation in practice.

*Keywords:* Single bottleneck model; Tradable credit scheme; Heterogeneous commuters; Market loss aversion; System optimal.

# Introduction

Urbanization is the rapidly increasing globally. In the past two hundred years prior to 1950 it is reported that around 400 million individuals moved to urban spaces. This number is expected to grow to more than 6 billion by the year 2030. Also urban areas will account for 70% of the world’s population in the next 10 years. (**Challenges and way forward in the urban sector**). This transition of population distribution to urban locations will result in many challenges including energy, safety, pollution and mobility.

As urbanization continues we will witness increased strain on our transportation infrastructure in manifold dimensions. Traffic congestion is one of the main challenges in urban set up which has been a pressing issue. The issue continues to grow as 88% of American own a car in order to save travel time. **(Why Traffic Congestion Is Here To Stay…And Will Get Worse)**. Another consequence of higher number of automobiles on the roads in the need for parking spaces. As the city area gets more and more expensive to live individuals start moving on the peripheries and hence that results in longer commuting time. Public transport in most places is inadequate or the network coverage is not good enough for individual to give up the comfort of a car. Increase in motorized traffic results mobility impacts on pedestrians and bicyclists for whom there is not enough consideration. One of the other main challenges on urban transportation is high maintenance cost of upgrading the infrastructure and also construction costs due to expansion. Higher number of automobiles on the roads also results in increasing levels of pollution which has adverse health impacts on the inhabitants. The area covered by roadways in urban areas can range from 30% to 60% which is really high. **(Urban Transport Challenges)**.

In order to tackle the issue related to urban mobility is to provide Mobility on Demand systems which provides the individual with mobility but can also provide the mobility other individuals without increasing congestion. Currently, a car is parked 95% of the times. This not only results is high level of under utilization of the vehicle but also leads to vehicle wear vis corrosion, which takes place when the vehicle is standing still. **(Today’s Cars Are Parked 95% of the Time)**. These systems are a response to the ever evolving demand of the population for more efficient transportation systems. These systems can be defined as services which provide mobility to individuals as a service. The advantages of these services is that the individuals do not have to have ownership related issues/costs for example insurance, depreciation etc [1]. MODs are also a more sustainable solution to the ever increasing traffic and congestion problem. The mobility on demand services have grown in the past and continue to see a positive response from the users. In fact industry research says that the car sharing market will increase to USD 16.5 billion by 2024. **(Car sharing Industry)**

The initial business model started with the services which centered around round trip type of service. For example Zip Car provides rental cars at stations but they need to be returned to the point it was rented. As the rental industry evolved with the mobile technology there are many services which provide one way rental/transportation like Alamo, Enterprise and Budget. These systems provide the flexibility of using the vehicle and not having to replace them from the point of rental. However, one of the issues of this service is that certain destinations are more in demand as compared to other destination. Hence as they system evolves over time we see that vehicles distribution is imbalanced over the geography. Some of the solutions which are proposed to tackle this issue is to incentivize customers to share rides or by hiring individuals to rebalance the cars. Both these approaches result in higher cost for the customers.

With the advent of robotic/autonomous vehicles there soon will be emergence of autonomous MODs are robotic vehicles providing transportation services. The issue of rebalancing then would be solved more efficiently as in the case of autonomous vehicles there will the vehicles can rebalance itself after the trip is completed.

In this new area potential managers of these AMOD systems have some questions which need to be addressed. Which locations need to be rebalanced? How much will rebalancing improve the systems performance? What policies should be used to rebalance? What will be the impact of rebalancing in a stochastic network with uncertain travel times? This papers attempts to solve this problem using an optimization framework.

This paper includes six sections. The introduction presents the background and motivations of this study. The literature review section illustrates its state-of-the-art methodology. Section 3 features the methodology. The findings and insights are discussed in the estimation results section. Finally, the study is summarized in the conclusion section.

# Literature review

Even though the study of rebalancing of AMOD systems itself is a relatively new area of research, but it has some common themes with one-to-one pickup and delivery problems (PDPs), characteristized by the absence of a central depot. PDPs can be dynamic or static. The DPDPs are further divided into dynamic stacker crane problem (Dynamic SCP), dynamic vehicle routing problem with pickup (Dynamic VRPPD) and deliveries and dynamic dial-a-ride problem. In dynamic SCP the vehicle transport only one unit from the pick site to the delivery location. This is mainly due to the capacity constraint of the vehicle. In dynamic VRPPD the number of items that can be carried is more than one but usually there are time windows for pickup and delivery. In dynamic dial-a-ride problem individuals can ask for a ride that need to be transported from one point to another. There are restrictions in this model on the time windows and the maximum length of the ride travel time. Rebalancing of AMOD systems differs from the DPDPs in having finite number of pickup and delivery locations, the destination of the customer is unknown and the optimization is done for the empty vehicles.

Rebalancing AMODs also have similarity to DTA wherein the time dependent flows are optimized over the network. The key point of deviation is the point where in rebalancing AMODs the empty trips are minimized instead of the passenger carrying trips.

Dimitris Papanikolaou, 2011 studied the problem of rebalancing using diffusion model. He modeled the rebalancing problem as diffusion of resource from areas of high concentration to areas of lower concentration. In this study though he did not account for the asymmetry in customer arrival. Similarly there have been other simulation based studies which looked at the rebalancing problem ( 11-14). The studies indicated that the car sharing systems are sensitive to the vehicle-to-trip ratio and the rebalancing scheme utilized. Also these simulation-based studies have significant number of parameters which need to be validated.

Later more studies [12-16] were done which had a more theoretical approach to solve the rebalancing problem. [12] anamyzed the problem from from fluidic approach where the the customer, vehicles are assumed as a continuum. In [8] the MODs are studied assuming human rebalancers using a queuing network approach. They applied the approach to the lower manhattan and showed that there is a need of 3-5 vehicle to driver ratio. These studies do provide added insights into the problem of rebalancing. There are a few gaps in these studies which we look to fill. The above works assumed that customers do not leave the system and the travel times are deterministic. In this paper we are going to adapt the fluidic approach to solve the rebalancing problem. This paper attempts to study the impact of stochastic travel time and customer leaving have on the rebalancing policy.

There are a number of studies that have looked at the issue of rebalancing [3], [4] where the total time spent by the empty vehicles in minimized on the network. [4] Looks at the problem as a fluidic system whereas [5] looks at it a queuing network model. [6] Looks at the problem by studying the effects of the rebalancing on the congestion by taking into account vehicle to vehicle interaction in a capacitated road network.

The severity of traffic congestion has significantly increased in metropolitan areas in recent decades. Commuters experience significant traffic delays during their morning and evening peak-period commutes. In the United States, they incurred an extra thirty eight hours of delay during peak periods annually in 2011 compared to 1982 (Schrank et al., 2012). Market-based instruments are proposed as efficient mechanisms to manage traffic demand. In the context of traffic congestion mitigation, they can be classified into price-based and quantity-based instruments.

A price-based instrument, widely known as congestion pricing, was first proposed by Pigou (1920). It has been implemented in different forms, ranging from congestion pricing on the San Francisco-Oakland Bay Bridge to Singapore’s area licensing scheme. Vickrey (1969) developed the simple deterministic bottleneck model to describe traffic congestion during the morning peak period where commuters pass a route segment with fixed capacity (bottleneck). In Vickrey’s model, commuters choose their departure times to reduce their own travel costs including schedule delay and travel time costs. Commuters cannot reduce their travel costs by unilaterally changing their departure times at user equilibrium (UE). Arnott et al. (1990) proposed a system optimal (SO) time-varying tolling scheme where the total system travel cost, which includes the costs of schedule delay and travel time, is minimized during the morning peak period.

Several efforts have sought to improve the realism of Vickrey’s and Arnott’s models by relaxing assumptions, in terms of flexible work starting times (Daganzo, 1985; Mun and Yonekawa, 2006), elastic demand (Arnott et al., 1993; Braid, 1989; Mun, 1994), multi-step tolling (Knockaert et al., 2016; Laih, 2004, 1994; Lindsey et al., 2012) and heterogeneous commuters in terms of schedule delay penalty, travel time penalty and desired arrival time (Doan et al., 2011; Newell, 1987). These studies develop continuous-time analytical formulations to determine the departure rates of commuters under the UE and SO conditions. By contrast, Ramadurai et al. (2010) proposed a linear complementarity formulation for the single bottleneck model in a discrete time setting. They prove the existence and uniqueness of a solution assuming both homogeneous and heterogeneous commuters in terms of schedule delay penalty, travel time penalty and desired arrival time. An advantage is its ability to solve for multiple heterogeneous commuter groups, unlike previous studies using continuous time setting. For example, Daganzo (1985) investigated managing morning commute congestion with only two groups, and Lindsey (2004) demonstrated the existence of equilibrium solution with heterogeneity of commuters without providing an algorithm to obtain the equilibrium solution. Along this line of research, Doan et al. (2011) constructed a linear complementarity formulation to investigate the effect of a time-varying tolling scheme on commuter’s departure rate. They formulated the SO condition as linear program and determined the SO group-agnostic time-varying tolls using linear programming duality.

Despite several studies on congestion pricing, it has been sparsely implemented in practice due to public resistance. For example, an official petition in 2007 collected 1.8 million signatures in the U.K. to end plans for a national charging scheme (de Palma and Lindsey, 2011). Travelers tend to view toll as another de facto flat tax imposed by the central authority. Further, if no compensation is provided under congestion pricing, it benefits only those who weigh the reduced delay more than the paid toll. Thereby, a significant portion of travelers may be worse off under congestion pricing (Hau, 1998; Lindsey, 2006). Hence, congestion pricing is perceived as an inequitable policy. These issues illustrate challenges with deploying congestion pricing schemes in practice. To overcome the aforementioned issues, some studies (Guo and Yang, 2010; Rouwendal et al., 2012) suggest rewarding travelers either using the revenue generated through tolls or subsidy from the central authority. However, even if the revenue of congestion pricing is redistributed, the central authority is the sole toll collector, and its claim of revenue-neutrality is difficult to verify and believe by the public (Yang and Wang, 2011).

Quantity-based instruments, such as a tradable credit scheme (TCS), have recently been suggested to resolve the issues inherent to price-based instruments. They have been extensively investigated for air pollution control by the European Union in energy-intensive sectors such as oil, paper, and power generation (Böhringer et al., 2009). In the transportation sector, Verhoef et al. (1997) propose the “tradable road-pricing smart card” to manage road transportation externalities. Akamatsu (2007) investigates the notion of time- and link-specific tradable permits to manage traffic congestion in a road bottleneck during the pre-specified peak period. Since the permits are time-specific, this scheme is able to eliminate traffic congestion by issuing the number of permits equivalent to the bottleneck capacity. However, this scheme entails a number of issues. First, travelers need to acquire link-specific permits along their chosen path before departure, where the price of permits is link- and time-specific. Second, the central authority needs to establish several trading markets. Finally, the central authority sells the permits in an auction market to travelers. Hence, it cannot resolve the issue of wealth transference from travelers to the central authority.

To overcome the complexity of the tradable permit scheme, Yang and Wang (2011) propose the tradable mobility credit scheme where a central authority determines the TCS parameters including credit allocation and charging schemes. The credit allocation scheme is characterized by the total endowment of travelers and the method of credit allocation. The central authority also determines the credit charging scheme, i.e., the subsequent link tolls to charge travelers. Thereby, travelers need to pay credits to be able to travel in the network. Travelers can also trade credits among themselves in the market. It is demonstrated that the traffic and market equilibrium conditions depend on the total number of allocated credits, and are hence independent of the credit allocation scheme. While any congestion pricing has a TCS mirror that is equally effective in managing congestion (Nie and Yin, 2013), there are potentially fewer social objections to TCS due to two reasons. First, it does not entail transfer of wealth between the central authority and travelers. Second, travelers with less value of time are directly compensated for higher experienced travel time by their ability to sell credits to those with higher value of time. Hence, tradable credit scheme is a promising equitable alternative to congestion pricing to reduce public resistance.

Numerous efforts have sought to investigate the effects of TCS on managing morning commute congestion. Tian et al. (2013) investigate the modal split of morning commuters between auto and transit under an optimal TCS implementation. Nie and Yin (2013) propose a credit allocation scheme for homogeneous commuters, in which the planning horizon is divided into peak and off-peak periods. Commuters are rewarded for traveling during the off-peak period, and are charged credits by the central authority for traveling during the morning peak period. Xiao et al. (2013) examine the effects of the initial credit allocation scheme on equity and welfare aspects of TCS, in the context of morning commute congestion. While various studies investigate TCSs for managing bottleneck congestion, they address only the homogeneous case or are limited to special heterogeneous cases (such as heterogeneity in travel time penalty). For example, while Nie and Yin (2013) and Tian et al. (2013) focus on the SO design of TCS, the former assumes commuter homogeneity and the latter considers heterogeneity only in terms of travel time penalty with a continuous cumulative distribution function. Similar to Tian et al. (2013), Xiao et al. (2013) assume commuter heterogeneity only in terms of travel time penalty, with a continuous cumulative distribution function to investigate the UE condition under a given TCS, and the SO design of TCS. However, for practical realism, it is essential to factor commuter heterogeneity in terms of schedule delay penalty, travel time penalty and desired arrival time to investigate TCSs for managing bottleneck congestion since they affect the departure time choices and credit consumption of commuters.

This study focuses on managing morning commute congestion during the peak period using a TCS. In contrast with previous studies, the UE and SO conditions are formulated by considering commuter heterogeneity in a discrete time setting using complementarity constraints. This approach has two advantages: (1) it offers a framework to study the existence and uniqueness of the UE solution under TCS, and (2) the equilibria can be computed efficiently using Lemke’s algorithm (Lemke, 1965). As empirical studies suggest that commuters have different sensitivities to travel and schedule delay costs based on socioeconomic characteristics (Small, 1982), commuter heterogeneity in terms of travel time penalty, schedule delay penalty and desired arrival time, is considered to better understand the effect of the TCS implementation. In the proposed TCS, commuters are grouped based on their value of time, schedule delay penalty and desired arrival time. The central authority implements a group-specific credit allocation scheme where a predetermined number of credits are allocated free of cost to each commuter of a group, based on a predetermined credit allocation method by group. In other words, each group of commuters receive different initial credit endowments compared to other groups to use during morning peak period. Then, the central authority determines a time-varying group-specific credit charging scheme in which commuters of a group use a certain number of credits based on their departure times and group index. In other words, each group of commuters may pay a different number of credits to depart in each time interval compared to other groups under the group-specific credit charging scheme. It is assumed that the total number of credits are sufficient to address the commuters’ credit needs. Given the credit allocation and charging schemes, commuters can trade credits in the market with negligible transaction fees. They either sell credits to gain monetary benefit or purchase credits in the market by incurring monetary loss, based on their credit endowment and credits required to fulfill their travel needs.

A commuter’s departure time choice for the morning peak period depends on the travel disutility, which in this study includes travel time cost, schedule delay cost and credit consumption disutility. Commuters are assumed to have full information on the time-varying group-specific credit charging scheme for the morning peak period; hence, their departure time choice is assumed to be riskless. However, departure time choice involves commuter behavior towards monetary loss and gain associated with trading credits in the market. Kahneman and Tversky (1979) propose that people perceive the payoffs as gain or loss by benchmarking to some reference points. They demonstrate the effect of loss aversion behavior in the decision-making process when people treat losses and gains asymmetrically (Tversky and Kahneman, 1991). Traveler loss-aversion behavior has been studied using cumulative prospect theory (Tversky and Kahneman, 1992) to model route choices of travelers under travel time uncertainty (Avineri, 2006; Gao et al., 2010; Xu et al., 2011). In the context of managing traffic congestion with TCS, Bao et al. (2014) formulate a reference-dependent UE model using the value function (Tversky and Kahneman, 1991) to study the impact of travelers’ loss aversion behavior in route choice in the static context. Travelers choose to trade credits in the market by paying transaction fees. They compare the charged credits with a reference point, which is assumed to be their credit endowments. So, they perceive purchasing credits as loss and selling credits as gain. In this riskless decision-making process of trading credits, the market loss aversion behavior of travelers is considered which implies that loss looms larger than gain for them. However, they neglect the impact of the credit allocation method under TCS, and do not analytically illustrate the relationship between credit price and market loss aversion behavior of commuters. Our study follows a similar approach to model the gain and loss perception of commuters in trading credits in the market where it is assumed that the commuters perceive travel time and scheduled delay costs as a pure loss. In other words, commuters consider the absolute value of time and scheduled delay costs in the departure time choice.

The contributions of this study are fivefold. First, this is the first study to consider the heterogeneity of commuters, in terms of value of time, schedule delay penalty and desired arrival time, in deriving the equilibrium departure rates and credit price of commuters under a TCS. This enhances practical realism and enables the central authority to understand the market and commuter behaviors in practice. To do so, the study formulates the reference-dependent UE condition of commuters during the morning peak period as a linear complementarity problem (LCP) in a discrete time setting, where the travel disutility of commuters includes the schedule delay, travel time and credit consumption disutilities. Second, this study investigates the solution existence and uniqueness of the equilibrium departure rates and credit price under the proposed TCS. The uniqueness of equilibrium credit price is a sign of a healthy market. Without the price uniqueness, commuters have to purchase the required credits at uncertain prices, which can reduce their travel choices as credit price increases (Miralinaghi and Peeta, 2016). It is shown that the uniqueness of equilibrium credit price depends on the uniqueness of departure rates. Third, it is essential to understand the effect of market loss aversion behavior of commuters on the equilibrium credit price, as the TCS may become inactive if the market loss aversion behavior of commuters is significant. This would reduce the ability of the central authority to manage morning commute congestion. We analytically demonstrate the effects of commuters’ loss aversion behavior on the equilibrium credit price. Fourth, this study demonstrates the effect of credit allocation method on the equilibrium credit price. It is proved that if travelers are equally sensitive to monetary losses and gains, the credit allocation method does not impact the equilibrium credit price. The numerical results also demonstrate the effect of credit allocation method on the equilibrium credit price and departure rates. Finally, the SO TCS design is developed as a benchmark for planners to minimize the total system travel cost, which includes the costs of schedule delay and travel time, while incorporating the heterogeneity of commuters. The proposed TCS design determines the SO group-specific credit allocation and time-varying group-specific credit charging schemes by applying linear programming duality. The numerical results demonstrate that the central authority can achieve Pareto-improving SO TCS design, that makes everybody better off through the appropriate design of group-specific credit allocation schemes.

The remainder of the paper is organized as follows. Section 2 includes preliminaries for the single bottleneck model with heterogeneous commuters in a discrete time setting. Section 3 formulates the reference-dependent heterogeneous single bottleneck model in discrete time with a TCS. Also, it illustrates the proofs for solution existence and uniqueness. Section 4 develops a primal-dual method to determine the SO TCS design. Section 5 discusses the results of some numerical experiments. Section 6 provides some concluding comments.

# Preliminaries

As preliminaries, we provide a brief description of the analysis of the morning commute congestion in a discrete time setting, proposed in Ramadurai et al. (2010) and later extended in Doan et al. (2011). Consider a highway with a bottleneck connecting the residential area and workplace, where commuters travel from home to workplace during the morning peak period. Each commuter is assumed to travel in his/her own vehicle to his/her workplace. A certain number of commuters pass the segment of highway with limited deterministic capacity, , in a discrete peak period with time intervals indexed by . The set of time intervals is denoted by . Let denote the cardinality of commuter groups , where commuters are classified based on their value of time, schedule delay penalty and desired arrival time. A commuter of group incurs unit cost of travel time (expressed in $/(time interval)/(vehicle)). Though travel time includes free flow travel time and queuing delay, the former is assumed to be zero. Thereby, in this study, the terms “travel time” and “queuing delay” are used interchangeably.

Commuters of group have a desired arrival time interval of to their workplace, where they incur a “schedule delay cost” for arriving earlier or later. The travel demand of group during the morning peak period is denoted by . It is assumed that , and are strictly greater than zero. If commuters arrive earlier, they incur unit cost for early arrival. If they arrive later, they incur unit cost for late arrival (expressed in $/(time interval)/(vehicle)). Based on empirical studies (Small, 1982), it is assumed that . Commuters choose their departure times, , based on the total travel cost, which includes two components: (1) the travel time , and (2) the schedule delay. Travel time and schedule delay are expressed in units of time intervals, as the average value of time and schedule delay, respectively, over all commuters departing in time interval . The departure rate of commuters of group in time interval is represented by (expressed in (vehicles)/(time interval)). Upon arriving at the queue, vehicles are served in a first-in-first-out order. The travel time of commuters is given by (Ramadurai et al., 2010):

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

Equation (1) denotes the travel time of commuters departing in time interval . Equation (2) is a recursive function implying that if departure rates are greater than bottleneck capacity, queue builds up over time and accumulates until the bottleneck capacity clears the queue. The early arrival delay, of commuters of group departing in is determined as follows (Ramadurai et al., 2010):

|  |  |  |
| --- | --- | --- |
|  |  |  |

Equation (3) denotes that early arrival delay of commuters of group departing in is zero if they arrive late to the workplace. The travel cost, , of commuters (expressed in $/(vehicle)) of group departing in is as follows (Ramadurai et al., 2010):

|  |  |  |
| --- | --- | --- |
|  |  |  |

At equilibrium, no commuter can reduce his/her total travel cost by unilaterally changing departure time. Doan et al. (2011) prove that commuters experience zero queuing delay under the SO condition. Using this SO feature, they propose a time-varying group-agnostic congestion pricing strategy to minimize the total system travel cost. They formulate the SO model as the following linear model:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

where the dual variable of constraint ‎(6) is the SO time-varying group-agnostic toll.

# Reference-dependent user equilibrium under TCS

The proposed TCS is characterized by its initial credit allocation and charging schemes. The credit allocation and charging schemes are predetermined and credits are allocated to the commuters initially. Let denote the credit allocation scheme where the central authority allocates credits to each commuter of group Let denote the credit charging scheme, where is the number of credits charged for group commuters departing in . In this context, represents the TCS with initial group-specific allocation of credits and time-varying group-specific credit charging scheme . Let denote the total number of credits allocated to commuters, which is given by . Commuters trade credits in the market based on their initial credit endowments and travel needs for each unit of credit. In the competitive market, the credit price, , depends on the interaction between the credit and travel markets during the peak period. In the context of the managing morning commute congestion using TCS, the credit is a commodity generated by central authority, and its supply is predetermined prior to the peak period. The credit demand depends on the credit charging scheme and commuter travel demand. Since the credit supply and commuter travel demand are constant, credit price is assumed to be constant through the peak period (Xiao et al., 2013).

In the competitive market, commuters gain benefit by selling excess credits if their initial credit endowments are greater than the number of charged credits. However, if their initial credit endowments are not sufficient to address travel needs, commuters purchase required credits from the market, which is considered as a loss. If commuters value losses more heavily than gains, they have different behaviors in purchasing or selling credits in the market. In this study, the value function proposed by Tversky and Kahneman (1991) is applied to determine the outcome of trading credits in the market. It incorporates the following two features:

1. The value of outcome is defined as gain or loss compared to a reference point;
2. The value function is linear, and steeper for losses than gains to reflect that people are more sensitive to potential losses (Thaler et al., 1997).

In this context, the credit outcome is considered as gain by commuters if they sell excess credits in the market. Otherwise, it is considered as a loss. Thereby, the reference point for a group is the initial credit endowment for a commuter of that group. Then, the corresponding credit consumption disutility, , of a commuter of group departing in time interval is as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where parameter denotes the “loss aversion” coefficient, indicating that commuters are more sensitive to loss than gain. Let denote the gain of a commuter of group departing in time interval by selling excess credits in the market when credit charge is less than initial endowment. The monetary gain of selling excess credits can be obtained as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where if , and 0 otherwise. Equation (6) can be written as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Using this notation, equation (5) can be rewritten as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Under the TCS, commuters choose their departure times, , based on the total travel disutilities, which include three components: (1) queuing delay cost, (2) schedule delay cost, and (3) credit consumption disutility. The travel disutility, , of a commuter of group departing in time interval is as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Since the credit consumption disutility can be negative, the travel disutility of a commuter of group departing in time interval can be either negative or positive. A negative travel disutility implies that the monetary gain of a commuter by selling his\her excess credits in the market is higher than the summation of the experienced queuing delay costs and schedule delay costs. The mixed-linear complementarity problem (MLCP) for the equilibrium problem with TCS is as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Here, is the equilibrium travel disutility of group . Mathematically, means “perpendicular”, i.e., vectors if and only if . Constraint (10) is the complementarity condition to ensure the dynamic UE, labeled as the DUE, condition. The departure rate of commuters of group in time interval is positive, only if the sum of travel time cost, schedule delay cost and credit consumption disutility of commuters is equal to the equilibrium travel disutility of group . The travel time of commuters departing in time interval is computed using constraints (11) and (12). The early arrival cost of a commuter of group departing in time interval is determined using constraint (13). Constraint (14) computes the monetary gain of selling excess credits for a commuter of group departing in time interval . Constraint (15) denotes the travel demand conservation condition. Constraint (16) denotes the credit market equilibrium condition in which the equilibrium credit price is greater than zero only if commuters consume allocated credits during the peak period. In constraint (16), the total number of allocated credits, , is reformulated as .

The MLCP (10)-(16) is reformulated as a pure LCP to leverage existing theorems in the LCP context to investigate solution properties such as solution existence in terms of equilibrium departure rates, credit price and travel disutility. It is written as the equivalent linear complementarity problem (ELCP):

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | |  | |
|  |  | |  | |
| (11)-(14), (16) | |  | |  | |

where is a sufficiently large constant parameter. denotes the revised equilibrium travel disutility of group . The following theorem proves the equivalence between MLCP and ELCP.

**Theorem 1.** MLCP and ELCP are equivalent.

**Proof.** MLCP and ELCP are equivalent if every solution to the ELCP satisfies two conditions:

1. It has a strictly positive value for .
2. is equal to .

Condition ‎(i) ensures that the demand conservation constraint ‎(19) is satisfied. Constraint ‎(21) indicates that

|  |  |  |
| --- | --- | --- |
|  |  |  |

which verifies that is always greater than zero because it is the minimum value of the summation of large parameter and travel disutility of group departing at time , . Since is strictly positive, complementarity constraint ‎(22) implies that the travel demand is met exactly, and hence the demand conservation constraint ‎(19) is satisfied.

Condition (ii) indicates the relationship between the equilibrium travel disutility obtained from MLCP and the revised equilibrium travel disutility obtained from LCP. If this condition is satisfied, the equilibrium travel disutility in constraint ‎(14) is replaced by . To illustrate this relationship, the complementary constraint (10) indicates that

|  |  |  |
| --- | --- | --- |
|  |  |  |

Because is a constant parameter, equation (19) is written as follows

|  |  |  |
| --- | --- | --- |
|  |  |  |

Then,

|  |  |  |
| --- | --- | --- |
|  |  |  |

This concludes the proof.

# 3.1. Solution existence

In this subsection, the solution existence of MLCP is investigated following Ramadurai et al. (2010) who address the single bottleneck model for the case without TCS. The solution existence for MLCP can be established by applying a fundamental existence result for linear complementarity problem to ELCP. To simplify the analysis, ELCP is normalized by multiplying the right-hand side of the complementarity constraints (11) and (12) by . Also, equation (8) is reformulated to include it in constraint (10) as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

ELCP is equivalent to the following form of :

|  |  |  |
| --- | --- | --- |
|  |  |  |

In complementarity constraint (24), is a real matrix, is a vector in and superscript denotes the transpose of matrix. In the context of ELCP, the vector of variables in constraint (24) is as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where , , , and . In constraint (24), is the constant vector with the following form:

|  |  |
| --- | --- |
|  |  |

where **1** is a vector of ones. Matrix , partitioned in accordance with the vectors and is given by

|  |  |
| --- | --- |
|  |  |

where is the identity matrix and

The solution existence of has been investigated in different studies (Cottle et al., 1992; Cottle and Dantzig, 1968; Lemke, 1965). The solution set of is denoted by . Cottle et al. (1992) proved that if belongs to both of the following matrix classes:

(i) Let Then, is called an -matrix if The class of such matrices is denoted by .

(ii) is copositive if for every nonnegative vector .

**Theorem 2.** Let denote the feasible set of departure rates defined by has a solution if a TCS such that .

The proof of this theorem is given in Appendix A.

# 3.2. Uniqueness of equilibrium solution

In this section, we investigate the uniqueness of equilibrium credit price and travel disutility. Further, the effect of market loss aversion behavior of commuters on the equilibrium credit price is examined. The uniqueness of the equilibrium credit price is important to investigate as it is a sign of a healthy market. The uniqueness is described in the following theorem:

**Theorem 3.** Let , and denote the equilibrium travel time of commuters, early arrival delay of commuters, and monetary gain of selling excess credits by group departing in time interval , respectively. Given a TCS , the equilibrium credit price is unique if the following two conditions are satisfied:

(i) Equilibrium departure rates are unique.

(ii) There exists at least one group of commuters whose equilibrium departure rates are positive in at least two different time intervals with different credit charges during the peak period.

**Proof.** To prove the uniqueness of the equilibrium credit price, consider the case where commuters consume all allocated credits during peak period. Otherwise, constraint (16) becomes inactive and the equilibrium credit price is zero and unique. If there exist two time intervals and in which commuters of group depart, then:

|  |  |
| --- | --- |
|  |  |
|  |  |

Hence, the equilibrium credit price can be written as:

|  |  |
| --- | --- |
|  |  |

Combining equations (6) and (30) yields:

|  |  |
| --- | --- |
|  |  |

There are four possible cases related to and where . In the first case, . Then, equation (31) can be rewritten as follows:

|  |  |
| --- | --- |
|  |  |

In the second case, . Then, equation (31) can be rewritten as follows:

|  |  |
| --- | --- |
|  |  |

In the third case, . Then, equation (31) can be expressed as:

|  |  |
| --- | --- |
|  |  |

In the fourth case, . Then, equation (31) can be expressed as:

|  |  |
| --- | --- |
|  |  |

Since (i) implies unique departure rates, it can be concluded that the equilibrium credit price is unique in any of these four cases. Therefore, the equilibrium credit price is unique if departure rates of commuters are unique and strictly positive in at least two different time intervals during the peak period.

To provide an alternative expression of the equilibrium credit price, we sum constraint (10) over and . By combining this summation with equation (6) and constraint (15), it follows:

|  |  |
| --- | --- |
|  |  |

If , the equilibrium credit price cannot be derived using equation (36) because:

|  |  |
| --- | --- |
|  |  |

Hence, the equilibrium credit price is removed from equation (36). If , then equilibrium credit price is given by

|  |  |
| --- | --- |
|  |  |

where denotes the summation of the equilibrium schedule and queuing delay costs of commuters. Equation (38) is valid for both the cases in terms of whether commuters consume or do not consume all issued credits during the peak period. If they do not consume all issued credits, then the TCS does not impact the equilibrium cost of commuters, and . Therefore, the equilibrium credit price is equal to zero if commuters do not consume all credits during the peak period.

While the equilibrium credit price in equations (32), (33) and (34) depends on the market loss sensitivity of commuters, the equilibrium credit price under the fourth case in the proof of theorem 3 (equation (35)) is independent of commuters’ loss aversion coefficient . This is because the loss aversion coefficient does not impact the travel disutility of commuters of group in time interval if the initial credit endowment of group is higher than or equal to the charged credits in that period. Consequently, if the equilibrium credit price is solely determined by the departure rates of commuters whose initial credit endowments are higher than credit charges while commuters of other groups depart in only one time interval, then the loss aversion of commuters does not impact the equilibrium credit price. However, this special case would rarely occur in practice because commuters of different groups are likely to depart in more than one time interval. This special case is ignored in this study to analyze the effect of loss aversion behavior of commuters on equilibrium credit price and total value of traded credits in the market. In Proposition 1, the effect of market loss sensitivity of commuters is investigated on equilibrium credit price in the market.

**Proposition 1.** If the commuter sensitivity to loss increases, then the equilibrium credit price approaches zero.

**Proof.** If commuters’ sensitivity to loss increases, it follows that

|  |  |  |
| --- | --- | --- |
|  |  | |
|  | |

Equations (32), (33) and (34) can be substituted into equation (39). As can be seen in these equations, as the loss sensitivity of commuters increases, the equilibrium credit price approaches zero if and are bounded. Using equation (2), it follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Inequality (40) can be simplified as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

It follows that is bounded and less than or equal to . Hence, is less than or equal to . Further, it follows from equation (3) that and hence, is less than or equal to . Since and are bounded, the equilibrium credit price approaches zero as the loss sensitivity of commuters increases. This completes the proof.

Proposition 1 implies that if commuters’ sensitivity to monetary loss of purchasing credits in the market increases, then commuters change departure times such that they purchase less credits in the market. Therefore, credit demand in the market reduces while credit supply remains constant. Consequently, equilibrium credit price decreases as commuters’ loss sensitivity increases. This trend continues until equilibrium credit price approaches zero. If the equilibrium credit price becomes zero, then the TCS becomes inactive and the central authority fails to manage the morning commute congestion using the TCS. Since credit price approaches zero, the total value of traded credits in the market approaches zero. Therefore, we have the following proposition.

**Proposition 2**. Total value of traded credits in the market approaches zero as the loss sensitivity of commuters increases.

**Proof.** Since the number of traded credits in the market is finite and the equilibrium credit price approaches zero as commuters’ loss aversion increases, it follows

|  |  |
| --- | --- |
|  |  |

This concludes the proof.

The next proposition investigates the effect of the credit allocation scheme on credit price when commuters are equally sensitive to the monetary loss and gain associated with trading credits in the market.

**Proposition 3.** If commuters are sensitive to loss and gain equally, the total allocated credits are the only factor in the credit allocation scheme to determine departure rates and credit price. In other words, the credit allocation method does not impact the equilibrium departure rates and credit price.

**Proof.** If commuters are equally sensitive to loss and gain, then is equal to 1. Credit consumption disutility in equation (8) can be simplified as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |

As the number of allocated credits to commuters of each group is constant, it follows from equation (31) that the equilibrium credit price is independent of the credit allocation scheme when is equal to 1. Therefore, the credit consumption disutility can be further simplified as:

|  |  |  |
| --- | --- | --- |
|  |  |  |

However, the total number of allocated credits in constraint (16) affects the departure rates and equilibrium credit price. This concludes the proof.

Previous studies on managing traffic congestion using TCS assume that the initial credit allocation scheme does not impact the commuters’ departure time choices. This assumption is supported by Yang and Wang (2011) in which the optimality conditions are independent of initial credit allocation scheme. Proposition 3 also indicates that initial credit allocation scheme does not impact the departure rate of commuters if they are equally sensitive to losses and gains. The next theorem investigates the uniqueness of equilibrium travel disutility.

**Theorem 4.** Equilibrium travel disutility is unique if equilibrium credit price is unique.

**Proof.** To prove the uniqueness of equilibrium travel disutility, assume that equilibrium credit price is greater than zero. If it is zero, then TCS becomes inactive and so, model (10)-(16) reduces to UE formulation of no-toll morning commute congestion of Ramadurai et al. (2010). They also prove that equilibrium travel disutility is unique under no-toll morning commute congestion.

Given the equilibrium credit price , the equilibrium credit consumption disutility in equation (8) can be rewritten as

|  |  |  |
| --- | --- | --- |
|  |  |  |

Hence, the equilibrium credit consumption disutility is unique if the equilibrium credit price is unique. Then, MLCP can be rewritten as follows

|  |  |  |
| --- | --- | --- |
|  |  |  |
| (11)-(13), (15) |  |  |

Following the approach of Ramadurai et al. (2010) to prove the uniqueness of equilibrium travel cost in no-toll morning commute congestion, it can be proved that is unique. Because is unique, the equilibrium travel disutility is unique if the equilibrium credit price is unique. This concludes the proof.

# System optimal design of TCS

This section proposes a primal-dual method to obtain the SO TCS design in terms of the initial group-specific credit allocation scheme and time-varying group-specific credit charging scheme.

Doan et al. (2011) prove that, given the SO departure rates, the queuing delay of commuters is equal to zero, i.e. . As discussed earlier, Doan et al. (2011) propose the model ‎(5)-‎(8) to derive the SO departure rates of commuters and demonstrate that the dual variable for constraint ‎(6) is the group-agnostic toll to achieve the SO departure rates. Similar to the study of Doan et al. (2011), this study develops a primal-dual method to determine the SO TCS design where the SO departure rates are assumed to be given and derived from model ‎(5)-‎(8). First, the time-varying group-specific SO credit consumption disutility is derived and then, the SO TCS parameters can be calculated based on .

Mathematically, we want to find the SO credit consumption disutility, so that the SO departure rates is the solution of the model ‎(14)-‎(20) under SO TCS. Let’s define . If the set of equations ‎(52)-‎(58) holds, then and are the optimal solution of model ‎(14)-‎(20) based on the SO departure rates where the queuing delay is equal to zero.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Constraint ‎(52) satisfies constraints ‎(14) and ‎(17) where queuing delay is equal to zero. Constraints ‎(15) and ‎(16) are also satisfied since queuing delay is zero. Constraints ‎(54), ‎(55) and ‎(56) are identical to constraints ‎(19), ‎(18) and ‎(20), respectively. Constraint ‎(53) ensures that the equilibrium departure rates are identical to SO departure rates. To regulate the SO TCS parameters, it is sufficient to first determine the SO credit consumption disutility using equations ‎(52)-‎(56). Then, is used to determine the SO TCS parameters so that constraints ‎(12), ‎(57) and ‎(58) are satisfied. Using equations ‎(52)-‎(56), the SO model can be formulated as the following linear program:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

The objective function (59) is to minimize commuters’ total schedule delay cost. Constraint (60) restricts the departure rate of commuters of group departing in to be equal to the SO departure rates. Because of constraint (60), the feasible solution space contains only the desired SO departure rates. Because the feasible solution space only contains the desired SO solution, it has a unique solution in terms of departure rates. If and are Lagrangian multipliers for constraints (60)-(61), the dual problem of primal problem (59)-(62) is as follows:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
| , Unrestricted |  |  |

If constraint (60) is replaced by constraint (49), the SO credit consumption disutility is only time-varying which is a special case of this study. The next theorem provides a system of equations to determine SO credit price , group-specific credit allocation scheme , and time-varying group-specific credit charging scheme using the SO credit consumption disutility of the dual problem (63)-(65).

**Theorem 5.** Let be an optimal solution to the dual problem (63)-(65). Given the loss aversion coefficient of commuters, if satisfies the following set of equations:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

then ) are the SO credit price, and credit charging and allocation schemes.

**Proof.** It is sufficient to demonstrate that satisfy the model (10)-(16) where queueing delay is equal to zero. Using the duality theorem and principle of complementarity slackness, holds if which is equivalent to constrain (10). Since queuing delay is zero for the SO condition of the single bottleneck model, constraints (11) and (12) are also satisfied. Constraint (13) is implicitly accounted for by the cost of early arrival in . Constraints (66) and (67) ensure that constraints (14) and (16) are satisfied. SO departure rates satisfy the credit conservation constraint (15). Therefore, the optimal solution of the primal-dual problem ) is a feasible solution to UE problem (10)-(16). Then, if the central authority designs the TCS such that the equilibrium credit price, credit allocation scheme and credit charging schemes satisfy constraints (66) and (67), the outcome of TCS is the SO credit consumption disutility. This concludes the proof.

The system of equations (66) and (67) consist of equations and variables. Because the SO credit price can be set to be a constant (e.g. equal to one), the number of variables is reduced to variables in the system of equations (66) and (67). Since there are fewer equations than number of variables, the system of equations (66) and (67) is underdetermined. It either has no solution or has infinitely many solutions in terms of credit allocation and credit charging schemes. If the central authority implements the uniform credit allocation scheme, the number of variables reduce to which is equal to the number of equations. Then, the system equations (66) and (67) has either one or no solution under a uniform credit allocation scheme and a group-specific time-varying charging scheme. However, a well-formulated group-specific credit allocation scheme can improve the equity aspect of TCS while the central authority leverages both credit allocation and charging schemes to achieve SO. For example, it is demonstrated in numerical experiments in the next section that a group-specific credit allocation scheme enables the central authority to achieve Pareto-improving TCS in which every commuter is better off in terms of travel disutility. Hence, the central authority can consider other system objectives, such as equity, to determine the final SO TCS design.

# Numerical experiments

The numerical experiments seek to examine the effects of commuters’ schedule delay penalty, value of travel time, and desired arrival time on: (i) the UE departure rates, (ii) commuters’ travel disutilities, and (ii) total system travel cost under TCS. To do so, MLCP (10)-(16) is solved using GAMS (2015) and its NLPEC solver. Further, we investigate the effect of commuters’ loss aversion on: (i) the UE departure rates and credit price under TCS, and (ii) the SO design of TCS. To do so, the dual problem (63)-(65) and system of equations (66)-(67) are solved. In the experiments, the time duration is 100 time units and the bottleneck capacity is 10 vehicles per time unit. The parameters used are shown in Table 1. There are three groups with different values of , , , . Group 1 has flexible work hours but higher value of time compared to groups 2 and 3. Group 2 has more flexible work hours and higher value of time compared to group 3.

**Table 1.**

Input data.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Group |  |  |  |  |  |
| 1 | 35 | 10 | 26 | 70 | 100 |
| 2 | 27 | 11 | 28 | 65 | 200 |
| 3 | 20 | 13 | 29 | 60 | 500 |

The numerical experiments in Sections 5.1 and 5.2 are conducted for TCS schemes labeled TCS-1 and TCS-2, respectively. Section 5.1 illustrates sensitivity analyses of departure rates, travel disutility and total system travel cost under TCS-1 with respect to early arrival penalty and desired arrival time. TCS-1 is characterized by the uniform credit allocation scheme, and the group-specific time-varying credit charging scheme shown in Fig. 1. As shown in the figure, commuters of group 3 do not pay any credits to depart in any time interval. The total endowment of credits is 500, and they are uniformly allocated to commuters, that is each commuter receives 0.625 credits. Under this credit charging scheme, commuters of group 3 do not pay credits during the peak period. The TCS-2, adopted in Section 5.2, is characterized by the group-specific credit allocation schemes shown in Table 2 and the group-specific time-varying credit charging scheme shown in Fig. 1. Section 5.2 investigates the effects of the credit allocation schemes of TCS-2 on traffic and market equilibrium conditions. Under these credit allocation schemes, credits are uniformly allocated to only one group of commuters under each scheme. The total endowment of credits in all schemes is 500. Section 5.3 focuses on the SO TCS, unlike the other two which are suboptimal.

**Fig. 1.** The credit charging scheme for TCS-1 and TCS-2

**Table 2.**

Credit allocation schemes of TCS-2

|  |  |  |  |
| --- | --- | --- | --- |
|  | Scheme 1 | Scheme 2 | Scheme 3 |
| Group 1 | 5.0 | 0.0 | 0.0 |
| Group 2 | 0.0 | 2.5 | 0.0 |
| Group 3 | 0.0 | 0.0 | 1.0 |

# 5.1. Sensitivity analysis of user equilibrium condition under TCS-1

In this section, we investigate the impact of commuters’ schedule delay penalty and desired arrival time on the traffic and market equilibrium conditions under TCS-1. Fig. 2 shows the UE departure rates of commuters for the case in which the TCS is not implemented. The total system travel cost under the UE condition is equal to 476,937. The travel disutilities of groups 1, 2 and 3 are 505.62, 605 and 610.75, respectively. Because of high schedule delay penalty and low value of time, commuters of group 3 depart early in order to arrive at the workplace close to the desired arrival time. On the other hand, commuters of group 1 depart later in the peak period to avoid the queuing delay. We conduct sensitivity analysis to understand the effects of commuters’ early arrival penalty and desired arrival time of group 3 under TCS-1 on: (i) the UE departure rates, (ii) commuters’ travel disutilities, and (iii) total system travel cost. The loss aversion coefficient is assumed to be equal to 3.

**Fig. 2.** UE departure rates of commuters without TCS

Fig. 3 plots the departure rates of group 3, travel disutility and total system travel cost under different values of early arrival penalty of group 3. Fig. 3(a) illustrates that as the early arrival penalty of commuters of group 3 increases, they delay their departures to reduce the schedule delay cost. Then, the departure rates of group 3 significantly increase in intervals 20-26 since they have also higher late arrival penalty compared to other groups. Consequently, travel time increases in subsequent intervals, and hence some commuters of group 2 depart earlier to avoid queuing delay. Because some commuters of group 2 depart in intervals with fewer charged credits, the credit consumption of group 2 decreases which results in the decrease of market credit price until it reaches zero. As the credit price declines to zero, the TCS becomes inactive. Fig. 3(b) illustrates the travel disutility of commuters under different early arrival penalties of group 3. Though commuters of group 2 incur less credit consumption disutility as the early arrival penalty of group 3 increases, they incur higher travel time and schedule delay costs because of shift in departure times of group 3. Hence, the travel disutility of group 2 increases. Commuters of group 1 depart after interval 73 and incur less credit consumption disutility. Further, their travel times are not influenced by a shift in the departure rates of group 3. Hence, the travel disutility of group 1 reduces as the early arrival penalty of group 3 increases. Commuters of group 3 incur higher schedule delay cost and credit consumption disutility, which increases their travel disutility. Because of the increase in early arrival penalty of group 3 and the inactive TCS market to manage congestion, the total system travel cost increases which is shown in Fig. 3(c).

Fig. 4 shows the effect of the desired arrival time of group 3 on departure rates, travel disutility, and total system travel cost. Fig. 4(a) shows that as the desired arrival time of commuters of group 3 increases, they delay their departure times to avoid early arrival penalty. As commuters of group 3 delay their departure times to intermediate intervals of the peak period, commuters of groups 1 and 2 shift their departure times to earlier and later intervals of the peak period. Hence, the credit consumptions of groups 1 and 2 decrease, which leads to a reduction in credit price. Consequently, the credit consumption disutility of group 3 increases and the credit consumption disutilities of groups 1 and 2 decrease. Then, the travel disutility of group 1 decreases while the travel disutility of group 3 increases as seen in Fig. 4(b). Though the credit consumption disutility of group 2 decreases, travel disutility of group 2 increases because of higher schedule delay and travel time costs due to shift in departure times of commuters of group 1 to earlier time intervals. This trend continues until the credit price becomes equal to zero when the desired arrival time of group 3 is equal to 61. If the desired arrival time of group 3 is greater than 61, the credit price is zero and the TCS becomes inactive. As the desired arrival time of group 3 increases, commuters of group 3 delay their departure times, which reduces the travel time of the early and intermediate intervals of the peak period. Hence, commuters of groups 2 and 3 experience less travel time and schedule delay costs, which leads to reduction in their travel disutilites. On the other hand, commuters of group 1 experience higher travel disutility due to an increase in their travel time and schedule delay costs. Overall, as the desired arrival time increases to 61, the total system travel cost increases, and then reduces as the desired arrival time further increases because of the reduction in the total system travel time, as shown in Fig. 4(c).

(a) Departure rates of group 3

(b) Travel disutility

(c) Total system travel cost

**Fig. 3.** Departure rates, travel disutility, and total system travel cost for different values of early arrival penalty of group 3 under TCS-1

(a) Departure rates of group 3

(b) Travel disutility

(c) Total system travel cost

**Fig. 4.** Departure rates, travel disutility, and total system travel cost for different desired arrival times of group 3 under TCS-1

# 5.2. Impact of credit allocation scheme under TCS-2

In this section, we investigate the effects of credit allocation scheme and commuter’s loss aversion behavior on departure rates and credit price under TCS-2. Fig. 5 shows the effect of commuters’ loss aversion on credit price under the three credit allocation schemes, illustrated in Table 2. Because commuters of groups 1 and 2 have zero initial credit endowment under credit allocation scheme 3, they have to purchase credits in the market for their own travel needs, and hence credit demand is higher compared to schemes 1 and 2. Further, as commuters of groups 1 and 2 consume a portion of their allocated credits under credit allocation schemes 1 and 2 for their own travel needs, credit supply in the market is less compared to credit allocation scheme 3. The credit demand in the market is also lesser under credit allocation schemes 1 and 2 compared to scheme 3 because commuters of groups 1 and 2 consume a portion of their allocated credits under schemes 1 and 2. Despite lesser credit demand in the market, lesser credit supply leads to higher credit price under schemes 1 and 2 compared to scheme 3. Fig. 5 also validates that credit price approaches zero as commuters’ loss aversion increases. Further, it indicates that if commuters are equally sensitive to monetary losses and gains, then credit prices are identical under the three credit allocation schemes and equal to 22.41. This validates that if commuters are equally sensitive to monetary losses and gains, the credit allocation method does not impact credit price.

**Fig. 5.** Effect of commuters' loss aversion on credit price under the three credit allocation schemes

**Fig 6.** Effect of commuters' loss aversion and credit allocation schemes on the total value of traded credits in the market

Fig 6 shows the total value of traded credits in the market. It validates proposition 3 where total value of traded credits in the market approaches zero as the loss sensitivity of commuters increases. When commuters’ loss aversion coefficient is equal to one, credit price is identical under the three credit allocation schemes. Because group 3 receives and sells all credits in the market under the third credit allocation scheme, the total value of traded credits in the market is highest under credit allocation scheme 3, where commuters’ loss aversion coefficient is equal to one. As commuters’ loss aversion coefficient increases, the total value of traded credits under the third credit allocation scheme becomes higher compared to those under the other schemes despite the higher credit price under the second credit allocation scheme. This is because as the loss aversion coefficient increases, the difference between the credit price under schemes 1 and 2 reduces while the total number of traded credits is higher under the third credit allocation scheme in which group 3 receives and sells all credits in the market.

Fig. 7 demonstrates the effect of the three credit allocation schemes on travel disutility of groups 1 through 3 as commuters’ loss aversion increases from 1 to 25. It also compares the travel disutility for the cases with TCS and without TCS (which is labeled as NoTCS in the figure). Under the first scheme, group 1 is the sole recipient of credits. The group 1 commuters use the credit endowments to fulfill their travel needs and sell excess credits in the market. As illustrated by Fig. 7(a), they experience a lesser travel disutility under the first credit allocation scheme compared to second and third credit allocation schemes, and the NoTCS case. Under the third credit allocation scheme, commuters of group 3 cannot reduce their travel disutility by changing their departure times. Commuters of groups 1 and 2 also cannot reduce their travel disutilites by changing their departure times. Consequently, the credit price reduces proportional to the loss aversion sensitivity of commuters (equation (32)), and travel disutilities remain unchanged despite the increase in loss sensitivity. Hence, under this specific case, the loss sensitivity only impacts the credit price and does not impact the travel disutility of commuters. Fig. 7(b) shows that commuters of group 2 incur the least travel disutility under credit allocation scheme 2 where they are the sole recipient of credits. However, they experience the highest travel disutility under scheme 1 in which they need to purchase credits from group 1. Similar to Fig. 7(a), commuters’ loss aversion does not affect the travel disutility of group 2 under the third credit allocation scheme. Fig. 7(c) indicates that commuters of group 3 has lesser travel disutility under all credit allocation schemes compared to NoTCS since they are not required to pay credits to fulfill their travel needs, and groups 1 and 2 have to change their departure time to reduce their travel disutility. As commuters’ loss aversion coefficient increases, the credit price decreases, and hence the travel disutility of group 3 increases due to the higher credit consumption disutility. Fig. 7 also demonstrates that the loss aversion behavior of commuters can significantly change the travel disutility and, consequently, the departure rates of commuters.

Fig. 8 demonstrates commuters’ departure rate under credit allocation schemes 1 to 3 when loss aversion coefficient is 25. Because of the high loss aversion in purchasing credits, commuters aim to reduce credit consumption in the market by shifting departure times. As can be seen in Fig. 8(a), more commuters of group 2 depart earlier during the peak period under the first credit allocation scheme to avoid paying credits compared to the NoTCS case. Further, commuters of group 1 depart closer to their desired arrival time under the first credit allocation scheme compared to the NoTCS case. Fig. 8(b) illustrates that under the second credit allocation scheme, some commuters of group 3 start earlier to avoid paying credits. Fig. 8(c) shows that under the third credit allocation scheme, some commuters of group 2 start earlier to avoid paying credits while some commuters of group 3 depart later to avoid queuing delay.

(a) Travel disutility of group 1 under the three credit allocation schemes

(b) Travel disutility of group 2 under the three credit allocation schemes

(c) Travel disutility of group 3 under three credit allocation schemes

**Fig. 7.** Effect of commuters' loss aversion on commuters’ travel disutility under different credit allocation schemes.

(a) First credit allocation scheme

(b) Second credit allocation scheme

(c) Third credit allocation scheme

**Fig. 8.** Effect of commuters' loss aversion on commuters’ departure rates under different credit allocation schemes.

# 5.3. System optimal TCS

In this section, we first illustrate the importance of considering the effect of commuters’ heterogeneity, in terms of value of time, schedule delay penalty and desired arrival time, to determine the SO credit allocation and charging schemes. Then, we illustrate the effect of loss sensitivity of commuters on the SO design of TCS. Fig. 9 shows the SO departure rates of commuters. Under the SO condition, the total departure rate of commuters is equal to the bottleneck capacity, and hence commuters experience zero travel time. The total system travel cost under the SO condition is equal to 232,400. While some commuters of group 2 depart earlier during the peak period, all commuters of group 1 depart later in the peak period.

**Fig. 9.** Departure rate of commuters under the SO condition

To illustrate the importance of considering the effect of commuters’ heterogeneity, it is assumed that the central authority designs the SO TCS such that commuters are homogeneous in these three dimensions. Commuters’ value of time, early arrival penalty, late arrival penalty and desired arrival time are assumed to be equal to 23.625, 12.125, 28.375, and 62.5, respectively. These values are the weighted average of parameters shown in Table 1. The loss aversion coefficient of commuters is assumed to be equal to 5. Fig. 10 shows the SO credit charging scheme where commuters receive 15 credits and the credit price is set to 20. The SO total system cost is equal to 272,690 and travel disutility of commuters is equal to 472. If the travel time and schedule delays are calculated based on the heterogeneity of commuters in terms of values of time, schedule delay penalties and desired arrival times, the total system travel cost increases to 278,570. It can also be seen in Fig. 10 that the slopes of credit charging scheme change at time intervals 23 and 78 with the number of charged credits equal to 15. This is due to the loss sensitivity of commuters where they consider charged credits greater than 15 as monetary loss. Consequently, the credit charging scheme have lower slopes in intervals 23-78 because commuters’ loss sensitivity increases their credit consumption disutility.

Fig. 11illustrates the system optimal credit charging schemes under the heterogeneity of commuters. If the central authority implements the uniform credit allocation scheme, the SO credit charging scheme is presented in Fig. 11(a) where commuters of each group receive 15 credits. The total system travel cost is equal to 232,400 and travel disutilities of groups 1, 2 and 3 are equal to 754, 616 and 327, respectively. Hence, if the central authority factors the heterogeneity of commuters (in terms of value of time, schedule delay penalty and desired arrival time) to design the TCS, the total system travel cost can be reduced compared to the case where commuters are assumed to be homogeneous. Given the credit charging scheme in Fig. 11(b), the central authority can achieve the Pareto-improving SO TCS design that makes everyone better off by leveraging an appropriate group-specific credit allocation scheme. In this scheme, if the central authority allocates 16, 14 and 17 credits to each commuter of groups 1, 2 and 3, respectively, the travel disutilities of groups 1, 2 and 3 are equal to 480.7, 596, and 351, respectively.

**Fig. 10**. System optimal credit charging scheme with homogeneous commuters

(a) System optimal credit charging scheme under the uniform credit allocation scheme

(b) System optimal credit charging scheme under the group-specific credit allocation scheme

**Fig. 11.** System optimal credit charging schemes with heterogeneous commuters

(a) Credit allocation scheme

(b) SO credit charging scheme with

(c) SO credit charging scheme with

**Fig. 12.** System optimal credit allocation and credit charging schemes under different commuter loss aversion behaviors.

Fig. 12 illustrates the SO credit allocation and charging schemes. To study the effect of commuters’ loss aversion on TCS design, two cases are considered, with commuters’ loss aversion coefficient equal to 1 and 20 in cases 1 and 2, respectively. Fig. 12 shows the credit allocation and charging schemes for the two cases. The SO group-specific credit charging schemes enable the central authority to achieve SO departure rates using TCS depicted in Fig. 9. For example, the central authority charges lesser number of credits from group 2 to depart in intervals 8 to 12, so that some of commuters of group 2 motivate to depart in those time intervals. If the central authority implements the group-agnostic credit charging scheme and charges other commuters similar to group 2 in those time intervals, other commuters may also depart in those time intervals. Then, the central authority cannot achieve the SO departure rates using TCS depicted in Fig. 9.

In both cases, the credit price is set to 20. The total allocated credits in case 1 is 22700, while 23762 credits are allocated in case 2. The percentage of traded credits in the market decreases from 28.4% in case 1 to 10.88% in case 2. In case 1, travel disutilities of the three groups are 430.49, 329.26 and 247, respectively. As can be seen, this SO design of the TCS is also Pareto-improving where commuters are better off compared to the NoTCS case. In case 2, the travel disutilities of groups 1 and 2 are 307.037 and 20.653, respectively. However, commuters of group 3 incur a significantly higher travel disutility (2362.39) compared to NoTCS case, as they purchase credits in the market from commuters of groups 1 and 2. It demonstrates that if loss aversion behavior of commuters is not considered, the SO design of TCS can result in an inequitable scheme where one group incurs a significantly higher travel disutility compared to the travel disutility under the NoTCS case.

# Concluding comments

This study develops an analytical formulation for the management of morning commute congestion using TCS under commuter heterogeneity. It contributes to the literature by considering commuter heterogeneity and the effect of commuters’ loss aversion. The existence and uniqueness of the equilibrium departure rates, travel disutility and credit price are investigated. The effects of initial credit allocation scheme, total endowment of credits and method of credit allocation are analyzed while considering commuter loss aversion behaviors in trading credits. It is proved that as commuter sensitivity to loss increases, credit price and total value of traded credits approach zero. It is also demonstrated that if commuters are equally sensitive to loss and gain, the credit allocation method does not impact the equilibrium departure rates and credit price. Finally, a primal-dual formulation is developed to derive the SO credit allocation and charging schemes.

Numerical experiments are performed for three groups of commuters with different values of travel time and schedule delay penalties. Sensitivity analyses are conducted for departure time choice, credit price and travel disutility under different credit allocation schemes with respect to commuters’ loss aversion behavior. Numerical results validate that credit price and total value of traded credits approach zero as commuters’ loss sensitivity increases. It is demonstrated that while the total number of allocated credits remains constant, the credit allocation method impacts the credit price and travel disutility of commuters, and consequently it affects the departure rates of commuters. The SO design of TCS, in terms of credit allocation and charging schemes, is computed under different commuter loss aversion coefficients. It is observed that if loss aversion behavior is not considered, the SO TCS design results in an inequitable scheme where one group incurs a significantly higher travel disutility.

The study findings suggest that market loss aversion behavior of commuters affects their departure rates, the credit price and total value of traded credits in the market. Hence, the central authority must factor the commuters’ loss aversion in the design of TCS to ensure that commuters do not experience very high travel disutilities which would lead to public opposition in practice. Further, the initial credit allocation method influences the credit price and departure rates when commuters’ loss aversion behavior is considered in trading credits. It provides flexibility for the central authority to design the TCS compared to the case where equilibrium condition is independent of the initial credit allocation method and the credit charging scheme is the only tool meet objectives such as equity.

This study can be extended in several directions in the future. First, the proposed model can be extended to determine equitable design of TCS. A central authority can address the equity issues associated with implementing the TCS by designing group-specific credit allocation schemes and time-varying group-specific credit charging schemes. Second, a continuous time-varying group-specific credit charging scheme can be difficult to implement for managing morning commute congestion. Hence, a “step credit charging scheme” can be determined in which the credit charging scheme is constant over a few time intervals within the peak period. The discontinuity of the number of charged credits at the end of each time interval leads to the discontinuity of departure patterns of commuters. Different behavioral assumptions are proposed to model the discontinuity of departure patterns such as mass arrival (Arnott et al., 1990), separated waiting (Laih, 2004, 1994), and braking-induced idling (Lindsey et al., 2012). A future research direction is to analyze the effect of the TCS with a step credit charging scheme in a discrete time setting under the aforementioned behavioral assumptions. Third, this study only considers the loss sensitivity of commuters toward monetary gains and losses of trading credits in the market. Another future research direction is to consider the loss sensitivity of commuters related to desired arrival time. Fourth, it is assumed that travel demand during the morning peak period is fixed. Another future research direction is to extend the proposed model to the elastic travel demand case where travel demand is a function of travel disutility. In this case, credits are allocated to both revealed and latent commuters. Finally, the central authority is assumed to provide initial credit endowments for travelers under TCS. A future research direction is to investigate the effect of TCS on managing morning commute congestion when the central authority sells all credits to travelers through a competitive bidding process in an auction market.

# Acknowledgments

This work is based on funding provided by the U.S. Department of Transportation through the NEXTRANS Center, the USDOT Region 5 University Transportation Center. The authors are solely responsible for the contents of this paper.

# Appendix A. Proof of Theorem 2

This appendix presents a proof for Theorem 2. In this theorem, the assumption of ensures that total supply of credit is sufficient to address the travel need of commuters. It is sufficient to show that is both -matrix and copositive. First, it is shown that is a copositive matrix. Since is a positive definite matrix, the matrix can be decomposed as the sum of a positive semi-definite matrix and a non-negative matrix **,** where

and

For any given positive vector, it follows that:

|  |  |
| --- | --- |
|  |  |

where

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

By using equation (68), is written as follows:

|  |  |
| --- | --- |
|  |  |

Equation (69) can be simplified using the following equations:

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

Then, can be rewritten as follows:

|  |  |
| --- | --- |
|  |  |

Finally, can be obtained as follows:

|  |  |
| --- | --- |
|  |  |

Hence, is a copositive matrix. Second, we need to prove that is -matrix. To do so, it is sufficient to show that the zero vector is the only solution of the . In other words, if and only if:

|  |  |  |
| --- | --- | --- |
|  |  |  |

If **,** it yields that **,** and **.** To prove the “only if” condition, assume that **,** and **.** Then,

|  |  |
| --- | --- |
|  |  |

Since andare positive definite matrices, it follows that are zero vectors and credit price is equal to zero. This implies that is also a zero vector. So, is both a -matrix and copositive. Hence, has a solution.

# References

11. D. Papanikolaou. A new system dynamics framework for modelling behavior of vehicle sharing

systems. Proc. of Symposium on Simulation for Architecture and urban design, 2011.

12. J. Barrios. On the performance of flexible carsharing: a simulation-based approach. j, 2012.

13. M. Barth and M. Todd. Simulation model performance analysis of a multiple shared vehicle

system. Transportation Research Record, 1999.

14. A. Kek, R. Cheu, and M. Chor. Relocation simulation model for multiple-station shared-use vehicle systems. Transportation Research Record, 2006.

16. K. Treleaven, M. Pavone, and E. Frazzoli. Asymptotically optimal algorithms for one-toone

pickup and delivery problems with applications to transportation systems. IEEE Trans.

Automatic Control, (9):2261–2276.

17. M. Pavone, S. L. Smith, E. Frazzoli, and D. Rus. Int. Journal of Robotics Research.

18. S.L. Smith, M. Pavone, M. Schwager, E. Frazzoli, and D. Rus. Rebalancing the rebalancers:

Optimally routing vehicles and drivers in mobility-on-demand systems. In American Control

Conference, 2013.

15. D. Efthymiou, C. Antoniou, and Y. Tyrinopoulos. Spati

Akamatsu, T., 2007. Tradable network permits: A new scheme for the most efficient use of network capacity. JSCE J. Infrastruct. Plan. Manag. 63, 287–301.

Arnott, R., de Palma, A., Lindsey, R., 1990. Economics of a bottleneck. J. Urban Econ. 27, 111–130. doi:10.1016/0094-1190(90)90028-L

Arnott, R., Palma, A. de, Lindsey, R., 1993. A Structural model of peak-period congestion: a traffic bottleneck with elastic demand. Am. Econ. Rev. 83, 161–179.

Avineri, E., 2006. The effect of reference point on stochastic network equilibrium. Transp. Sci. 40, 409–420. doi:10.1287/trsc.1060.0158

Bao, Y., Gao, Z., Xu, M., Yang, H., 2014. Tradable credit scheme for mobility management considering travelers’ loss aversion. Transp. Res. Part E 68, 138–154. doi:10.1016/j.tre.2014.05.007

Böhringer, C., Rutherford, T.F., Tol, R.S.J., 2009. THE EU 20/20/2020 targets: An overview of the EMF22 assessment. Energy Econ. 31, S268–S273. doi:10.1016/j.eneco.2009.10.010

Braid, R.M., 1989. Uniform versus peak-load pricing of a bottleneck with elastic demand. J. Urban Econ. 26, 320–327. doi:10.1016/0094-1190(89)90005-3

Cottle, R.W., Dantzig, G.B., 1968. Complementary pivot theory of mathematical programming. Linear Algebra Appl. 1, 103–125. doi:10.1016/0024-3795(68)90052-9

Cottle, R.W., Pang, J.-S., Stone, R.E., 1992. The Linear Complementarity Problem. Academic Press Inc., Boston.

Daganzo, C.F., 1985. The uniqueness of a time-dependent equilibrium distribution of arrivals at a single bottleneck. Transp. Sci. 19, 29–37. doi:10.1287/trsc.19.1.29

de Palma, A., Lindsey, R., 2011. Traffic congestion pricing methodologies and technologies. Transp. Res. Part C Emerg. Technol. 19, 1377–1399. doi:10.1016/j.trc.2011.02.010

Doan, K., Ukkusuri, S., Han, L., 2011. On the existence of pricing strategies in the discrete time heterogeneous single bottleneck model. Transp. Res. Part B Methodol. 45, 1483–1500. doi:10.1016/j.trb.2011.05.019

Gao, S., Frejinger, E., Ben-Akiva, M., 2010. Adaptive route choices in risky traffic networks: A prospect theory approach. Transp. Res. Part C Emerg. Technol. 18, 727–740. doi:10.1016/j.trc.2009.08.001

Guo, X., Yang, H., 2010. Pareto-improving congestion pricing and revenue refunding with multiple user classes. Transp. Res. Part B Methodol. 44, 972–982. doi:10.1016/j.trb.2009.12.009

Hau, T.D., 1998. Road Pricing, Traffic Congestion and the Environment, in: Congestion Pricing and Road Investment. Edward Elgar, Cheltenham, UK, pp. 39–78.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47, 263–291.

Knockaert, J., Verhoef, E.T., Rouwendal, J., 2016. Bottleneck congestion: Differentiating the coarse charge. Transp. Res. Part B Methodol. 83, 59–73. doi:10.1016/j.trb.2015.11.004

Laih, C.-H., 2004. Effects of the optimal step toll scheme on equilibrium commuter behaviour. Appl. Econ. 36, 59–81.

Laih, C.-H., 1994. Queueing at a bottleneck with single- and multi-step tolls. Transp. Res. Part A Policy Pract. 28, 197–208. doi:10.1016/0965-8564(94)90017-5

Lemke, C.E., 1965. Bimatrix equilibrium points and mathematical programming. Manage. Sci. 11, 681–689.

Lindsey, R., 2006. Do Economists Reach A Conclusion on Road Pricing? The Intellectual History of an Idea. Econ J. Watch 3, 292–379.

Lindsey, R., 2004. Existence, Uniqueness, and Trip Cost Function Properties of User Equilibrium in the Bottleneck Model with Multiple User Classes. Transp. Sci. 38, 293–314. doi:10.1287/trsc.1030.0045

Lindsey, R.C., van den Berg, V.A.C., Verhoef, E.T., 2012. Step tolling with bottleneck queuing congestion. J. Urban Econ. 72, 46–59. doi:10.1016/j.jue.2012.02.001

Miralinaghi, M., Peeta, S., 2016. Multi-period equilibrium modeling planning framework for tradable credit schemes. Transp. Res. Part E Logist. Transp. Rev. 93, 177–198. doi:10.1016/j.tre.2016.05.013

Mun, S., 1994. Traffic jams and the congestion toll. Transp. Res. Part B Methodol. 28, 365–375. doi:10.1016/0191-2615(94)90035-3

Mun, S., Yonekawa, M., 2006. Flex time, traffic congestion and urban productivity. J. Transp. Econ. Policy 40, 329–358.

Newell, G.F., 1987. The morning commute for nonidentical travelers. Transp. Sci. 21, 74–88. doi:10.1287/trsc.21.2.74

Nie, Y., Yin, Y., 2013. Managing rush hour travel choices with tradable credit scheme. Transp. Res. Part B 50, 1–19. doi:10.1016/j.trb.2013.01.004

Pigou, A.C., 1920. The economics of welfare, 1st ed. Macmillan and Company, London.

Ramadurai, G., Ukkusuri, S. V., Zhao, J., Pang, J.-S., 2010. Linear complementarity formulation for single bottleneck model with heterogeneous commuters. Transp. Res. Part B Methodol. 44, 193–214. doi:10.1016/j.trb.2009.07.005

Rosenthal, R.E., 2015. GAMS — A User’s Guide. Washington, DC.

Rouwendal, J., Verhoef, E.T., Knockaert, J., 2012. Give or take? Rewards versus charges for a congested bottleneck. Reg. Sci. Urban Econ. 42, 166–176. doi:10.1016/j.regsciurbeco.2011.08.011

Schrank, D., Eisele, B., Lomax, T., 2012. 2012 urban mobility report. Texas Transportation Institute.

Small, K.A., 1982. The Scheduling of Consumer Activities: Work Trips. Am. Econ. Rev. 72, 467–479.

Thaler, R.H., Tversky, A., Kahneman, D., Schwartz, A., 1997. The effect of myopia and loss aversion on risk taking: an experimental test. Q. J. Econ. 112, 647–661. doi:10.1162/003355397555226

Tian, L.-J., Yang, H., Huang, H.-J., 2013. Tradable credit schemes for managing bottleneck congestion and modal split with heterogeneous users. Transp. Res. Part E Logist. Transp. Rev. 54, 1–13. doi:10.1016/j.tre.2013.04.002

Tversky, A., Kahneman, D., 1992. Advances in prospect theory: cumulative representation of uncertainty. J. Risk Uncertain. 5, 297–323. doi:10.1007/BF00122574

Tversky, A., Kahneman, D., 1991. Loss aversion in riskless choice: A reference-dependent model. Q. J. Econ. 106, 1039–1061.

Verhoef, E., Nijkamp, P., Rietveld, P., 1997. Tradeable permits: their potential in the regulation of road transport externalities. Environ. Plan. B 24, 527–548. doi:10.1068/b240527

Vickrey, W.S., 1969. Congestion theory and transport investment. Am. Econ. Rev. 59, 251–260.

Xiao, F., Qian, Z. (Sean), Zhang, H.M., 2013. Managing bottleneck congestion with tradable credits. Transp. Res. Part B Methodol. 56, 1–14. doi:10.1016/j.trb.2013.06.016

Xu, H., Lou, Y., Yin, Y., Zhou, J., 2011. A prospect-based user equilibrium model with endogenous reference points and its application in congestion pricing. Transp. Res. Part B Methodol. 45, 311–328. doi:10.1016/j.trb.2010.09.003

Yang, H., Wang, X., 2011. Managing network mobility with tradable credits. Transp. Res. Part B 45, 580–594. doi:10.1016/j.trb.2010.10.002

1. \* Corresponding author. Tel: +1 (765) 494-2209.

   *Email addresses*: smiralin@purdue.edu (M. Miralinaghi), peeta@purdue.edu (S. Peeta), Xiaozheng He hex6@rpi.edu (X. He), sukkusur@purdue.edu (S. Ukkusuri). [↑](#footnote-ref-1)